Enrollment No: ______________________ **Exam Seat No: ______________________**

C.U.SHAH UNIVERSITY

Summer Examination-2018

Subject Name: Real Analysis

Subject Code: 5SC02REA1 **Branch: M.Sc. (Mathematics)**

Semester: 2 **Date: 02/05/2018** **Time: 10:30 To 01:30** **Marks: 70**

Instructions:

(1) Use of Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

SECTION – I

Q-1 Answer the Following questions: **(07)**

a) Define: Measurable function **(02)**

b) If $A, B \subseteq R$ be such that $m^*(B) = 0$ then prove that $m^*(A \cup B) = m^*(A)$. **(02)**

c) If $A = \{a\}$ then find $m^*(A)$. **(01)**

d) What is measure of open sphere centre with (1,1) and radius 2 ? **(01)**

e) Define: Measurable set **(01)**

Q-2 Attempt all questions **(14)**

a) Prove that P is non-measurable set. Where P contains one element from each equivalence classes E_λ and $\cup E_\lambda = X = [0,1)$. **(09)**

b) If $E_1, E_2, ..., E_n$ be a finite sequence of measurable sets and they are mutually disjoint then for any $A \subseteq R$, $m^*\left(A \cap \left(\bigcup_{i=1}^{n} E_i\right)\right) = \sum_{i=1}^{n} m^*(A \cap E_i)$ **(05)**

OR

Q-2 Attempt all questions **(14)**

a) Let m be the set of all measurable subsets of R then prove that m is an algebra of R. **(07)**

b) Prove that outer measure of an interval is its length. **(07)**

Q-3 Attempt all questions **(14)**

a) Let A be an algebra on X and $\{A_i\} \in$ A then there exist $\{B_i\} \in$ A such that **(05)**

i) $\bigcup_{i=1}^{n} A_i = \bigcup_{i=1}^{n} B_i$ and ii) $B_i \cap B_j = \phi$, for $i \neq j$.

b) Let ϕ and ψ are simple functions on E which are vanish outside of a set of finite measure then prove that $\int a\phi + b\psi = a\int \phi + b\int \psi$. **(05)**

c) Let E_1 and E_2 be two measurable subsets of R then prove that $m(E_1 \cup E_2) + m(E_1 \cap E_2) = m(E_1) + m(E_2)$ **(04)**

OR

Q-3 Attempt all questions **(14)**

a) State and prove Bounded convergence theorem. **(07)**

b) Let f, g be two measurable function on *E*, where $E \in$ m then for any $f \cdot g$ are also measurable function on *E*. **(05)**

c) State Egoroff's theorem. **(02)**

SECTION – II

Q-4 Answer the Following questions: **(07)**

a) Find positive part of $f(x) = \frac{1}{2} + \sin x,\ 0 \leq x < 2\pi$. **(02)**

b) Define: Bounded variation **(02)**

c) If $f \leq g$ almost everywhere then $\int_E f \leq \int_E g$. **(02)**

d) What is difference between Riemann integral and lebesgue integral? **(01)**

Q-5 Attempt all questions **(14)**

a) State and prove monotone convergence theorem. **(07)**

b) State and prove Beppo-Levi's theorem. **(05)**

c) Write Chebychev's inequality. **(02)**

OR

Q-5 Attempt all questions **(14)**

a) Suppose $f, g \in BV[a,b]$ then prove the following: **(07)**

i) $fg \in BV[a,b]$ and ii) $\frac{f}{g} \in BV[a,b], where\ g \neq 0$.

b) State and prove Lebesgue dominated convergence theorem. **(07)**

Q-6 Attempt all questions **(14)**

a) Let f be a bounded measurable function on $[a,b]$ and $F(x) = \int_a^x f(t)dt + F(a)$ then $F'(x) = f(x)$ almost everywhere on $[a,b]$. **(07)**

b) Show that the function $f(x)$ is continuous but not bounded variation on $[0,1]$. **(05)**

Where $f(x)=\begin{cases} 0 & if\ x=0 \\ x\sin\left(\frac{\pi}{x}\right) & if\ 0<x\leq 1 \end{cases}$.

c) Prove that monotonically increasing functions are of bounded variation. **(02)**

OR

Q-6 Attempt all Questions **(14)**

a) State and prove Fundamental theorem of integral calculus. **(07)**

b) Prove that F is absolutely continuous function on $[a,b]$ if F is indefinite integral. **(05)**

c) Define: Indefinite integral **(02)**